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# **A FRAMEWORK FOR PROBABILISTIC ASSESSMENT OF FRACTURE MECHANICS LIFE (Preprint)**

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# A Framework for Probabilistic Assessment of Fracture Mechanics Life

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The paper develops a framework for application of probabilistic methods to assess the variability in fracture mechanics life for components. The probability of failure for a component at a specific failure location is typically defined by a crack size exceeding a threshold crack size (e.g. as defined by repair or bore-scope inspection limits). The computation of the probability of failure in a predictive manner requires the ability to compute crack initiation and crack propagation life in a probabilistic manner. This paper describes the essential elements of a framework that enables the computation of probabilistic fracture mechanics life and exercises the ingredients of this framework on a benchmark problem with focus on crack growth model parameter variability. The demonstration on the benchmark problem provides insights into some of the issues that need to be considered towards the development of a rigorous probabilistic life assessment framework. Some of the practical challenges towards extending the results to complex industrial applications are highlighted as scope for future efforts.

## I. Introduction

Probabilistic life analysis is a method used to evaluate the variation around life or time/cycles to failure predictions for components. Life predictions for components are done for a variety of reasons. Generally, these analyses fall into one of two categories. The first is assessment of the operational reliability (e.g. operational disruption such as unscheduled engine removal). The second category of analysis is durability (e.g. time to scrapping of the component). Those components that historically drive reliability or durability issues (e.g. hot section components) and whose lives are highly sensitive to variation are candidates for probabilistic analysis [1]. The probabilistic life assessment framework seeks efficient methods to characterize and propagate variation in the input variables to the life of various critical features. Figure 1 shows the process for deterministic assessment of component life at a particular failure location [2]. The development of a predictive probabilistic assessment framework for total life that incorporates all the elements of the deterministic process (e.g. component temperature, stress analysis, initiation life) is our long term vision.

Two key challenges in the development of such a probabilistic framework include (a) the ability of the deterministic life models to accurately represent the average life consumption of fielded hardware – uncertainty in our ability to predict life through models (e.g. assumptions on material microstructure effects, hold time effects, thermo-mechanical fatigue, crack tip plasticity effects) (b) the ability to characterize the various sources of variation that need to propagate through the life modeling process (e.g. heat transfer boundary conditions, crack initiation material models, creep material model variability etc.). The validation of such a predictive probabilistic life assessment framework requires the ability to obtain component field inspection data (e.g. presence or absence of cracks in fielded hardware, crack size of fielded parts). The life predictive models can be exercised in a probabilistic manner for the operational conditions that are representative of the component exposures and a schematic of a comparison between model predictive and field life is shown in Figure 2. Differences in the mean life between model and (good quality) field data provide an indication of the accuracy of the predictive life models and difference in standard deviation between models and (good quality) field data are representative of our ability to characterize and propagate the sources of variation through the life modeling process. The ability to predict both mean life and the variation in life accurately provides us with a better understanding of risk to failure

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for a variety of operating conditions on fielded hardware and provides a foundation towards design for reliability on new engine designs and architectures.

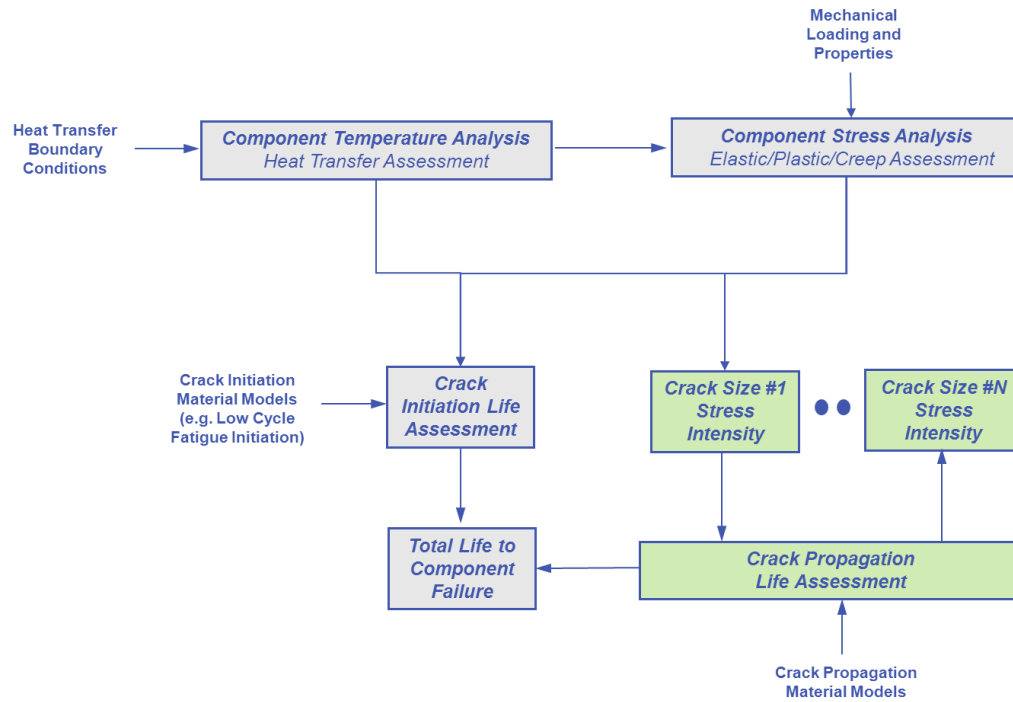


Figure 1: The process for deterministic assessment of component life at a failure location [2]

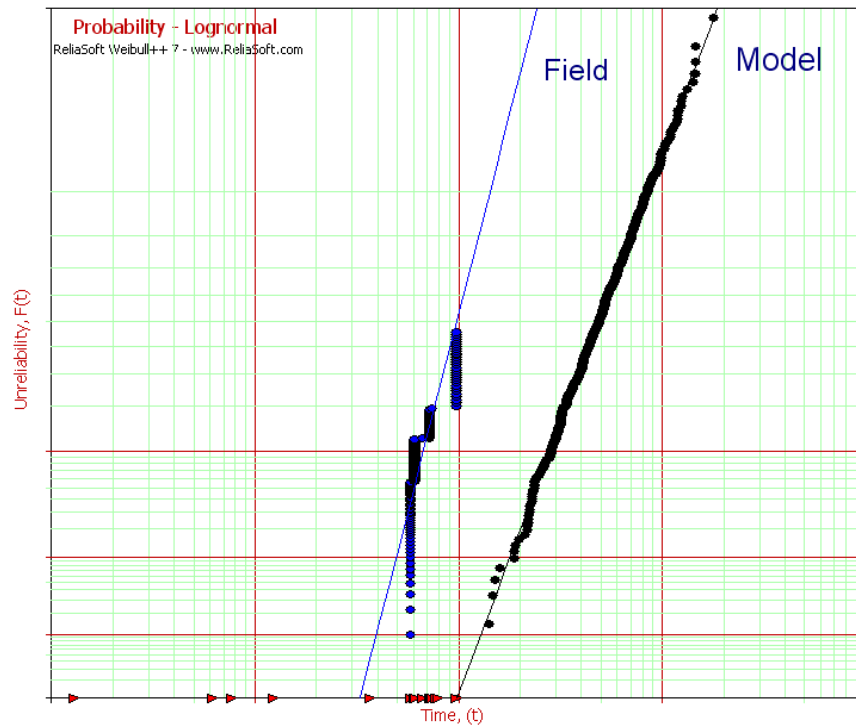
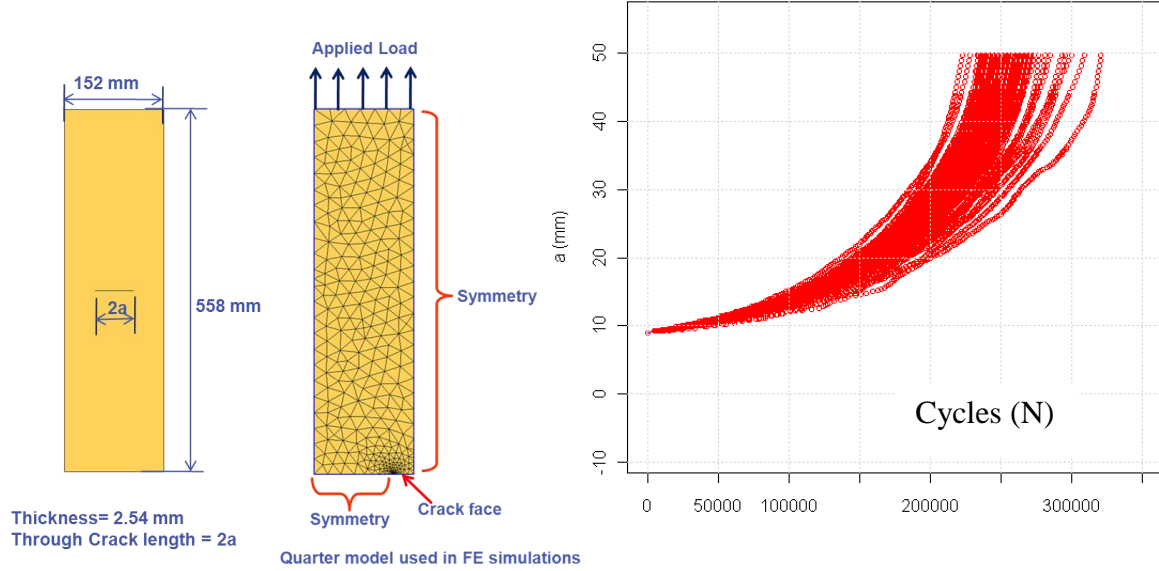


Figure 2: Schematic of an output from exercising a probabilistic predictive life assessment framework and validation with field life data at a specific failure location. Life to a crack reaching a threshold crack size is shown and is a combination of both initiation and propagation life.

The scope of the current paper is to take a first step towards the development of a probabilistic framework with emphasis on fracture mechanics life. A standard benchmark problem (referred to as “Virkler data” in literature) was provided by Air Force Research Labs with fixed mechanical loading at constant temperature for 2024-T3 Aluminum alloy [3]. The only significant source of variation that was considered was the crack growth model properties and the elements of the probabilistic framework that enable this benchmark are highlighted in this paper. Crack length ( $a$ ) versus cycles ( $N$ ) as shown in Figure 3 were provided for 68 center crack specimens under the same nominal loading conditions starting from an initial crack size of 9 mm to a final crack size of 49.8 mm, at a constant R-Ratio of 0.20 (with 164 measurements in intervals of 0.2/0.4/0.8 mm in different regions). This is also a good benchmark problem from the perspective that other authors have considered this in the recent past [4, 5].



**Figure 3: Crack length ( $a$ ) in mm versus cycles ( $N$ ) for 68 specimens from reference [3] shown here along with the finite element model used for stress intensity computations**

## II. Description of the Fracture Mechanics Benchmark

Examination of the data in Figure 3 reveals that there is large inter-specimen (specimen-to-specimen) variability in the fracture mechanics life (for a 9mm crack to grow to 48.2 mm) between 222792 to 320996 cycles. A secondary source of variability is the intra-specimen (within specimen) variation that can be observed through the wiggles in the crack growth trajectory of a single specimen. For the current effort, we will restrict our attention to the dominant inter-specimen variability. This is similar to the random variable approach described in reference [4] where an alternate random process/stochastic approach has also been considered to model the intra-specimen variability. The results from [4] also describe after detailed analysis that consideration of the inter-specimen variability is a viable approach to describe the variability in life for this problem. The model chosen to describe the crack growth behavior is the Paris model with parameters “ $C$ ” and “ $n$ ” that are random variables that vary from specimen to specimen.

$$\frac{da}{dN} = 10^C [\Delta K(a)]^n \quad (2.1)$$

This form of a model is consistent with that used in references [3, 5] and a higher order model has been used in reference [4] to describe the crack growth behavior. The relationship between the crack tip stress intensity range  $\Delta K$  and crack length is taken corresponding to a center crack specimen in [6], where  $\Delta P$  is the range of the applied load,  $W$  and  $t$  are the specimen width and thickness respectively,  $G(a)$  is the geometry factor associated with the center crack specimen.

$$\Delta K = \frac{\Delta P}{Wt} \sqrt{\pi a} G(a) \quad (2.2)$$

The following relationship was used for the geometry factor  $G(a)$ :

$$\begin{aligned} G_1(a) &= \sqrt{\frac{W}{\pi a} \tan\left(\frac{\pi a}{W}\right)} \quad ; \text{ if } 0 < \frac{a}{W} \leq 0.25 \\ G_2(a) &= \sqrt{\frac{1}{\cos\left(\frac{\pi a}{W}\right)}} \quad ; \text{ if } 0.25 < \frac{a}{W} \leq 0.4 \end{aligned} \quad (2.3)$$

Numerical integration of the following equation between appropriate limits was performed in order to get the non-linear relationship between  $a$  and  $N$  as described by the Paris model

$$\frac{da}{[\sqrt{\pi a} G(a)]^n} = 10^C \left[ \frac{\Delta P}{Wt} \right]^n dN \quad (2.4)$$

The values of  $W$  and  $t$  were 152.4 mm (6 in) and 2.54 mm (0.1 in) and the value of  $\Delta P$  was 18.682 kN (4.2 kips). The Paris constants “ $C$ ” and “ $n$ ” are computed in units consistent with the original paper [3] where units for stress-intensity  $\Delta K$  are in  $\text{ksi} \times \text{in}^{0.5}$  and units for  $da/dN$  are inches/cycles (A units conversion can be done to obtain Paris model “ $C$ ” in metric units where  $\Delta K$  is in  $\text{MPa} \times \text{m}^{0.5}$  and  $da/dN$  is in mm/cycles). The availability of a closed form  $\Delta K$  solution for the case of this load controlled crack growth situation eliminates the need to perform detailed 3D fracture mechanics finite element simulations, however the current framework is being developed to handle such scenarios in the future. Recent efforts in [7,8] have provided a generic interpretation of the Paris crack growth behavior that provides some insights into the high correlation coefficient between Paris coefficients observed in experimental testing and the specimen size dependencies of these parameters. The implications of some of this recent work on practical applications and on the predictive nature of these crack growth models (i.e. translating results from specimens to hardware, from one material pedigree/heat treatment to another) needs further study.

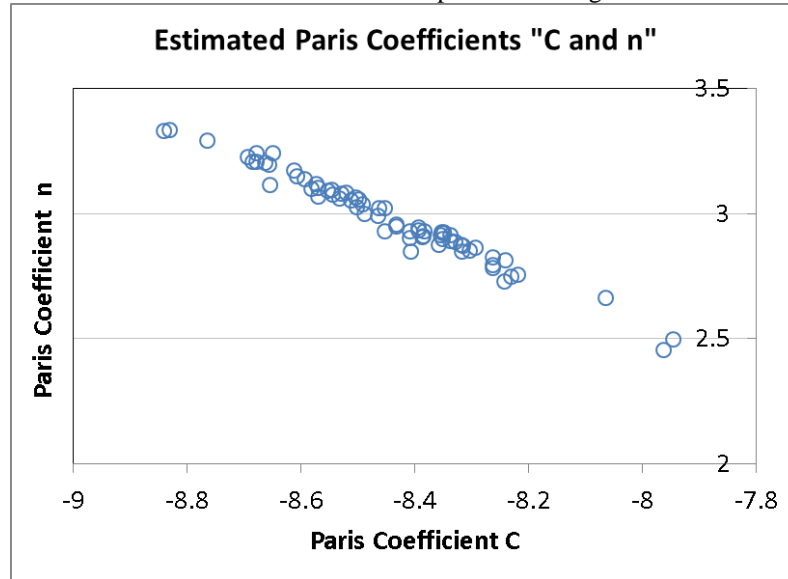
### III. Estimation of Crack Growth Model Parameter Variation

One can envision multiple approaches to characterize the crack growth model parameters  $C$  and  $n$  using data from the 68 specimens [e.g. 3-5]. We describe some options here:

1. Option 1: A non-linear estimator is used to estimate crack growth parameters for each specimen separately by matching “ $a$ ” versus “ $N$ ” information between the crack growth model (integrated form of equation 2.4) and data. A joint distribution for  $C$  and  $n$  is fit to the 68 estimated pairs. Such an approach is quick to develop, has been considered in reference [5] and lays the foundation for more advanced modeling methods (example: described in options 3 and 4)
2. Option 2: The crack growth rate versus stress intensity range follows a linear relationship in the log-log space according to Paris model and a linear regression model is used to infer crack growth model parameters. Such an approach is simple to implement but is dependent on the numerical scheme used to describe the crack growth rates from “ $a$ ” versus “ $N$ ” data (e.g. modified secant method has been proposed in reference [3] as a good choice). This option has been used in references [3-4] with reference [4] adopting a higher order crack growth rate model
3. Option 3: A non-linear mixed (fixed + random) effects model as described in [9] can be used to characterize the mean behavior of  $C$  and  $n$  (fixed effects) as well as the inter-specimen variability (random effects) in  $C$  and  $n$ . This is a more rigorous version of option 1 since it considers the likelihood of the model matching all 68 specimen data simultaneously to characterize the crack growth model parameters. This is a classical statistical reliability approach to handle repeated measures or degradation data and would fit well in the context of the current data [3]. The maximum likelihood computation is quite involved and this option will be considered in our future efforts.

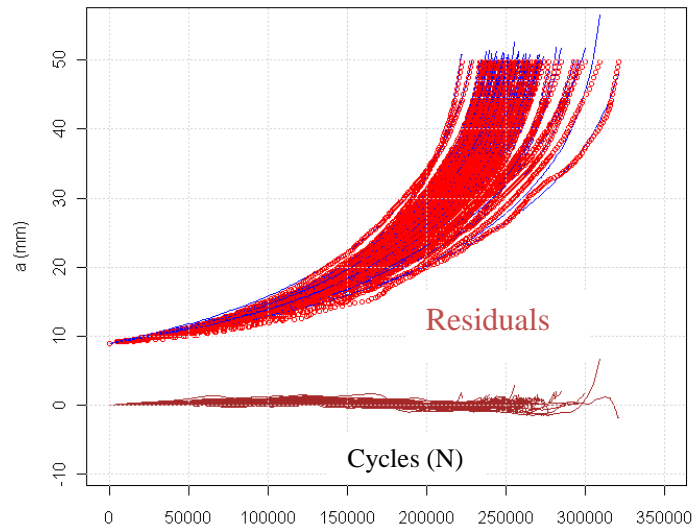
4. Option 4: Bayesian estimation methods would be good choices if we advance to more advanced crack growth models that capture curvature near the threshold and critical stress intensities. The crack growth data in this benchmark does not have sufficient information in these non-linear regimes (especially at the critical stress intensity) and the use of Bayesian methods that use other sources of 2024-T3 Aluminum data for threshold and critical stress intensity in conjunction with the data in [3] can be beneficial. Advanced crack growth models should also consider the implications of references [7, 8].

The current paper is focused on the comparison of the first two options listed above. For option 1, a non-linear least squares approach was used to minimize the model-data mismatch in crack length for various cycles at which the crack was observed [10]. The utilization of the non-linear least squares approach makes the “assumption” that residual errors (due to measurement uncertainty and modeling error) in the crack length are distributed in normal fashion. This assumption was tested at the end of the estimation process. The estimated Paris model coefficients for  $C$  and  $n$  are provided in Figure 4.

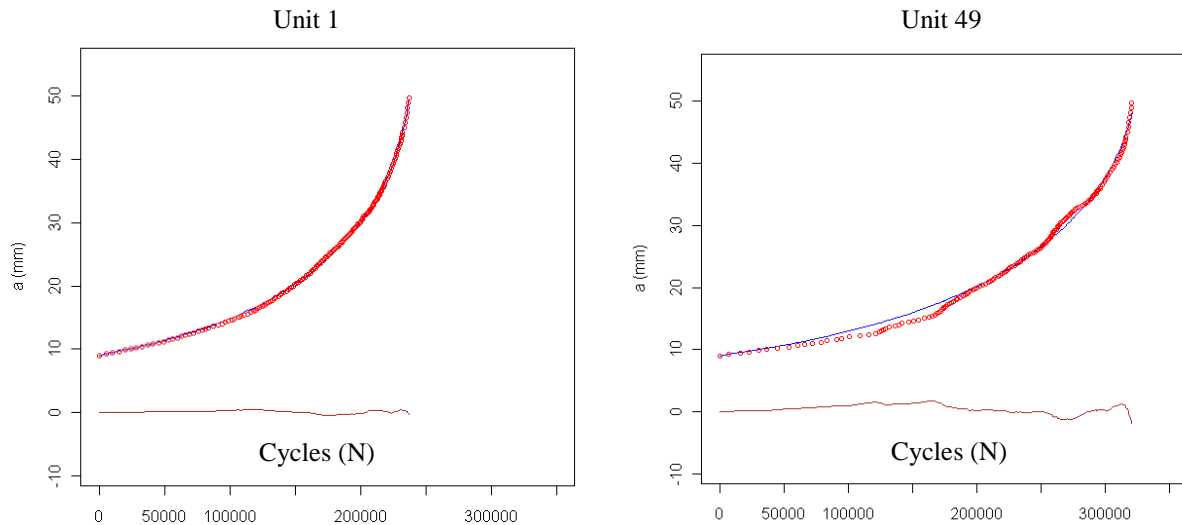


**Figure 4: Estimated Paris Coefficients using option 1 approach for 68 different specimens.**

The result of using the estimated  $C$  and  $n$  values in the crack growth model is shown against the backdrop of the data and along with the residuals in Figure 5. Results for 2 specific units (units 1 and 49) are shown in Figure 6 for better visual clarity.

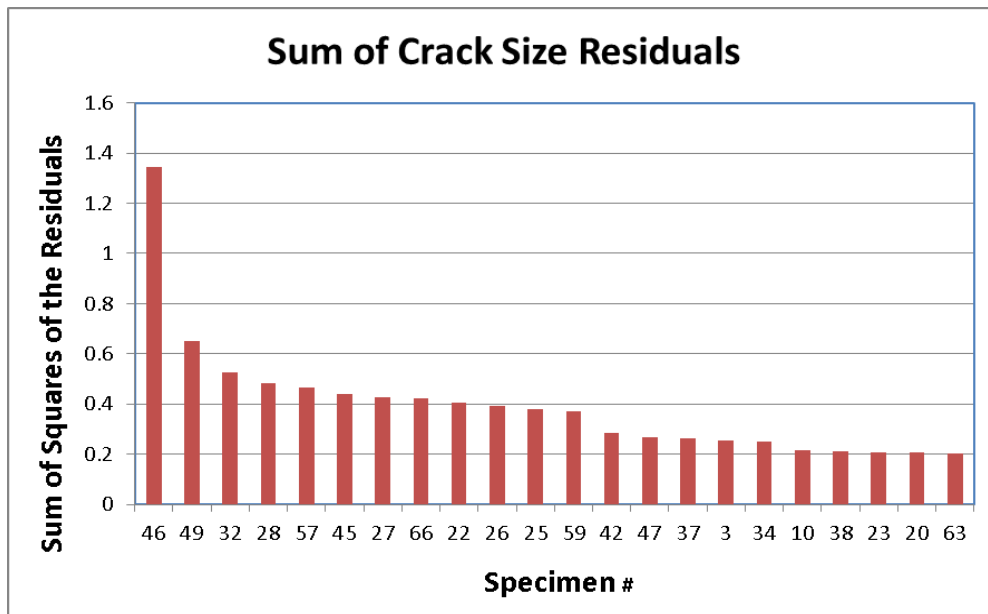


**Figure 5: “a” versus “N” for 68 specimens from the “estimated/fitted” model compared to the data. (Model shown in blue, data in red and residuals in brown)**



**Figure 6: “a” versus “N” comparisons for unit 1 and unit 49 highlighted for visual clarity**

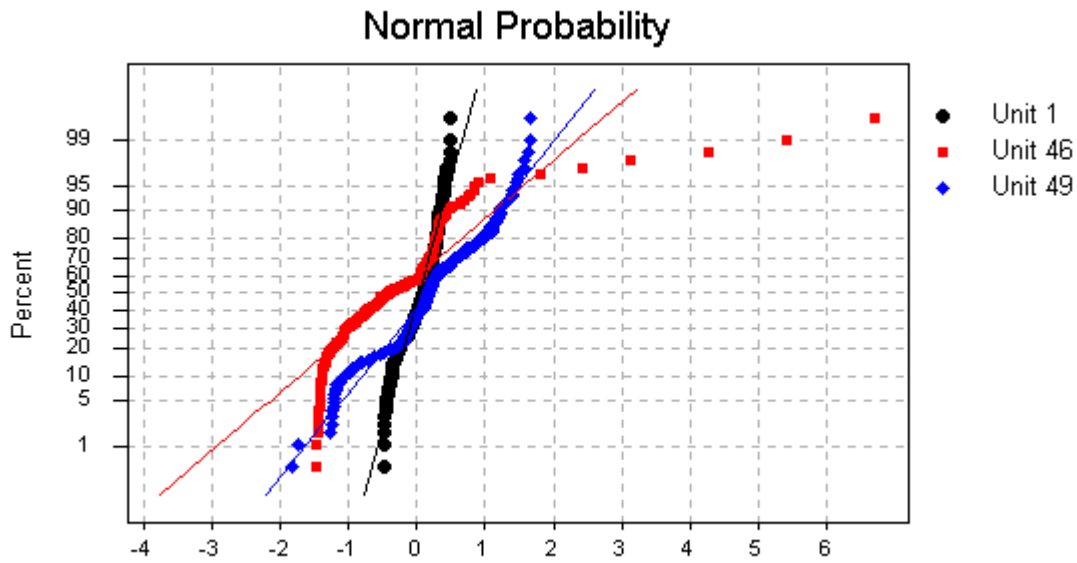
Based on these results, we infer that the inter-specimen variability in the crack growth model parameters is captured quite well. The intra-specimen variability (e.g. noticed in unit 49) where the model has a smooth behavior whereas the data shows some variation, is not captured as well. Figure 7 shows the sum of squares of the residuals (between the model and data) in crack size for various specimens. This measure provides a “quick” understanding of intra-specimen variability (i.e. local deviation from Paris model) that the experimental data demonstrates. This emphasizes the need to better characterize the local microstructure influence on the crack growth rates to improve life prediction ability.



**Figure 7: Sum of squares of residuals in the crack size between the model and the data for each of the specimens. Only specimens with higher residuals were shown here for clarity.**



Normal probability plots of residuals for units 1, 46 and 49 are shown in Figure 8. The better behavior of residuals and the consistent normality assumption for unit 1 relative to units 46 and 49 that had the highest two sum of squares of residuals is noted. Overall the normality assumption of the residuals (that permits the applicability of non-linear least squares) agrees quite well for most units with some strong deviations observed in units 46 and 49. A more comprehensive understanding of the intra-specimen variability due to the local/heterogeneous material microstructure is needed if we want to capture this aspect in the modeling process. Some of these aspects can be captured through the generalized Paris model forms suggested in [8].



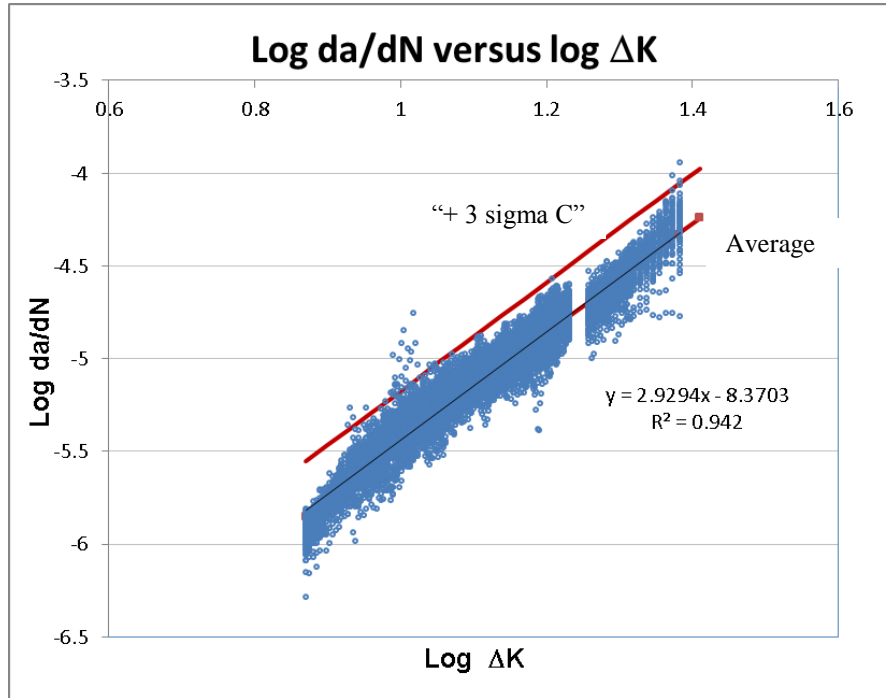
**Figure 8: Normal probability plot of crack size residuals between the fitted model and data for 3 representative units**

Estimates from option 2 (estimation using  $da/dN$  versus  $\Delta K$ ) were obtained in order to compare against the results from option 1. Figure 9 shows the  $da/dN$  (inches/cycle) versus  $\Delta K$  (ksi- $\sqrt{\text{in}}$ ) values for the 68 specimens in log-log scale. A modified secant method as suggested in [3] was used to compute the crack growth rates. It must be noted here that the crack size measurements were provided at 0.2/0.4/0.8 mm intervals (depending on the crack size region) and introduces a source of numerical error in the estimate of the crack growth rate derivatives. This results in a difference in accuracy between the approaches used in option 1 (or option 3) investigated in this work, reference [5] versus the approaches that were used in references [3, 4]. A least squares regression line over all the data points provides mean estimates for  $(C, n)$  as  $(-8.3703, 2.9294)$ .

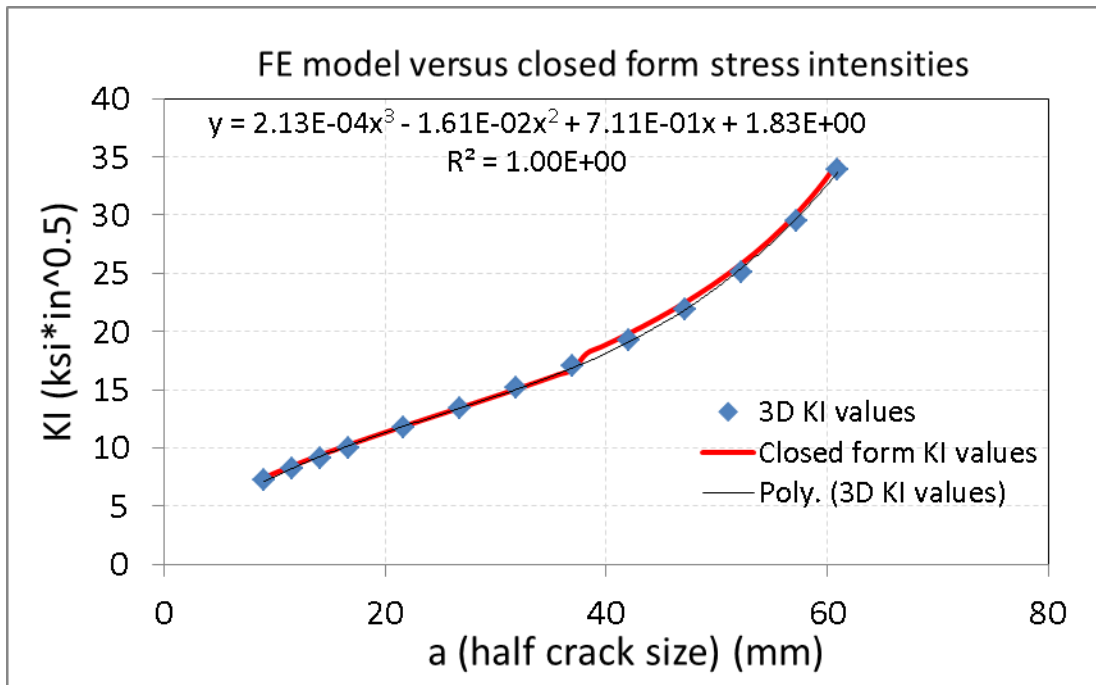
A quick engineering estimate of minimum crack propagation life is often obtained by using the “+3 sigma material properties” on the Paris intercept  $C$  and keeping the Paris slope  $n$  as a constant. It will be shown that such a method to compute the minimum life is not necessarily consistent with the actual “-3 sigma propagation life”. A more detailed description of the limitation of such approaches has been highlighted in reference [5].

The other observation that can be made through Figure 9 is that the crack growth data shows some amount of curvature near the threshold that the Paris model cannot capture. This would motivate the use of advanced crack growth models that captures the near threshold/short crack behavior more accurately. The break that is observed in  $\log \Delta K$  values at around 1.2 is due to the fact that we are using different geometry factors  $G(a)$  around a  $a/W$  ratio of 0.25. A comparison of the geometry factors from a finite element model versus the closed form solution is shown in Figure 10.

Figure 11 shows the comparison of “C and n” estimates from the two different estimation options for the 68 specimens. This plot shows the significantly different estimates that one obtains on a unit by unit basis for each of the specimens from the two options. One of the reasons for the difference is the numerical error in crack growth rate derivative computations in the option 2 method. Various options for the computation of the crack growth rate have been investigated in [3], but an effective way to eliminate this problem would be to choose the option 1 type approach to obtain the crack growth material properties. The option 1 approach is shown in section IV to improve the life computation accuracy compared to the option 2 approach.

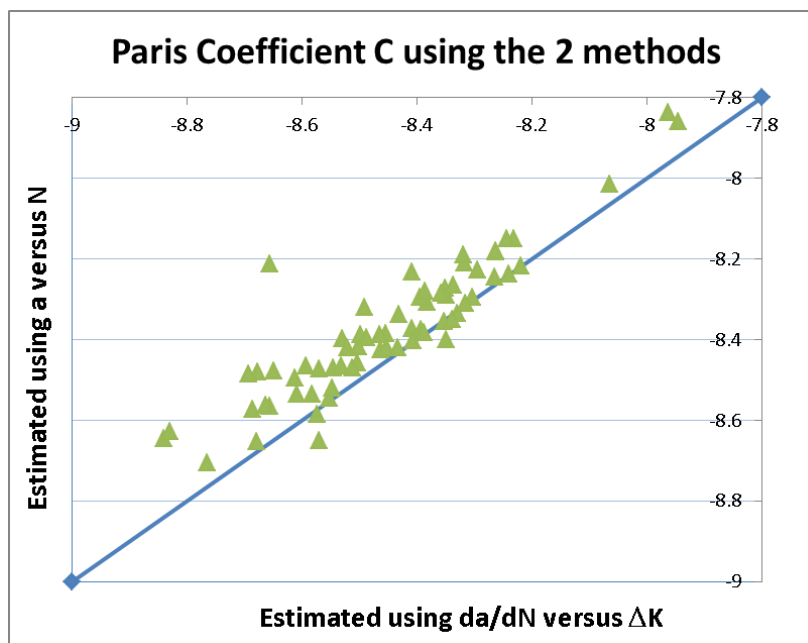


**Figure 9: “da/dN” versus “ΔK” for 68 specimens shown in Log-Log plot. “+3 sigma” value of C is shown to demonstrate a flaw in computation of “-3 sigma” life.**



**Figure 10: Comparison of the 3D finite element based crack stress intensity geometry factors versus the closed form geometry factors; showing the jump in the closed form solution at  $a/W$  of 0.25.**

The relationship between “ $da/dN$ ” versus “ $\Delta K$ ” corresponding to mean values of estimates from both options is shown in Figure 12. This plot shows that the crack growth rates are overestimated using the option 2 “ $da/dN$ ” versus “ $\Delta K$ ” method relative to the option 1 “ $a$  versus  $N$ ” method. This motivates the need to be sensitive about the computation of the crack growth rates from the raw experimental data while using the option 2 approach in estimating crack growth material properties and scatter. These findings are expected to be applicable even after we advance to other crack growth model forms (e.g. suggested in references [7,8]).



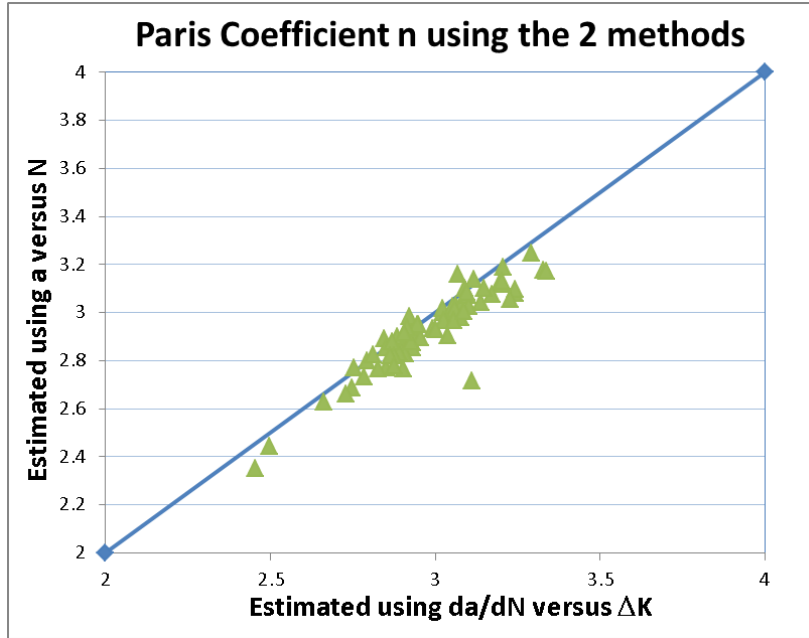


Figure 11: A cross plot of Paris coefficient estimates ( $C$ ,  $n$ ) from the 2 options for estimation is shown for each of the 68 specimens.

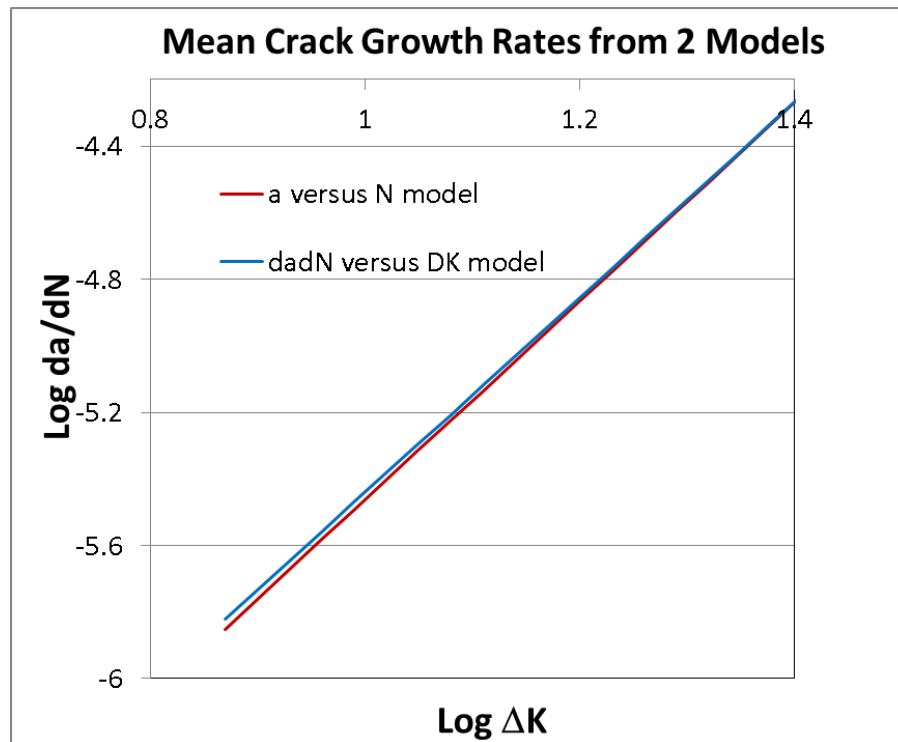
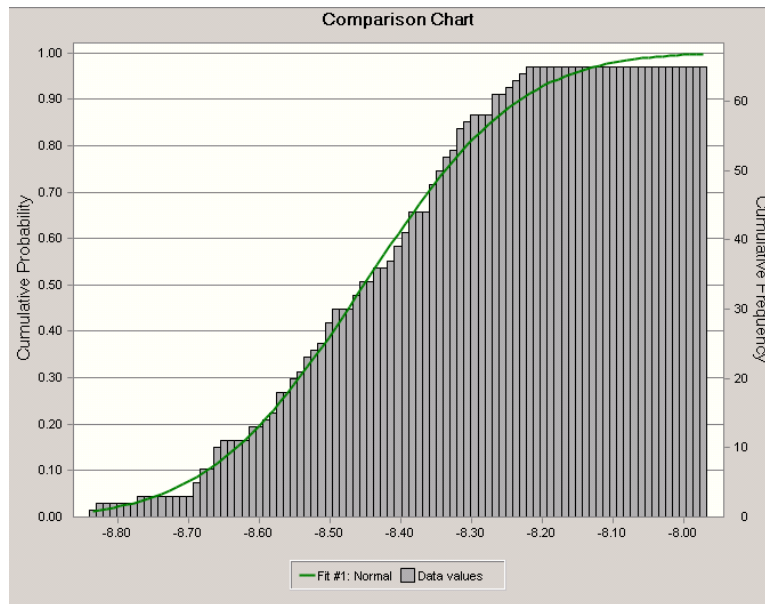
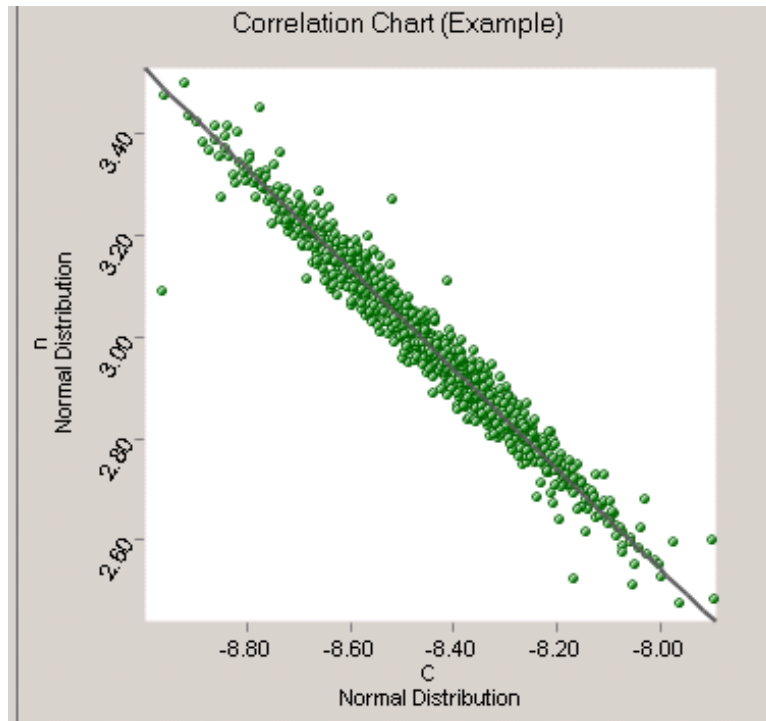


Figure 12: Mean crack growth rates estimated using option 1 and option 2 methods to characterize crack growth model coefficients.

#### IV. Probabilistic Assessment of Fracture Mechanics Life

In order to translate the impact of crack growth model parameters on fracture mechanics life and in order to leverage these results for future simulations – a joint distribution was computed for (C, n) using the 68 estimates obtained from option 1 and option 2 methods. A more comprehensive description of the benefits of using such a joint distribution has been discussed in reference [5]. This was performed using the Crystal Ball software and is shown in Figure 13. It was observed that the normal distribution did not perfectly characterize the variability observed in C, n. Improvements to capture this variability would include (a) the ability to fit multivariate distributions (b) understanding the impact of sample size and (c) understanding the limitation of Paris-law type model coefficients in describing the crack growth behavior, and these aspects will be pursued in subsequent efforts. Results reported in this paper assume a normal distribution for “C” and “n” and these provide a starting point to understanding and predicting the variability in fracture mechanics life. The current paper uses the traditional Monte Carlo analysis with around 5000 runs and the advanced mean value method (a type of fast probability integration method) from [7] to propagate the crack growth model parameter variability to compute the life distributions from the model and compare versus the coupon data.

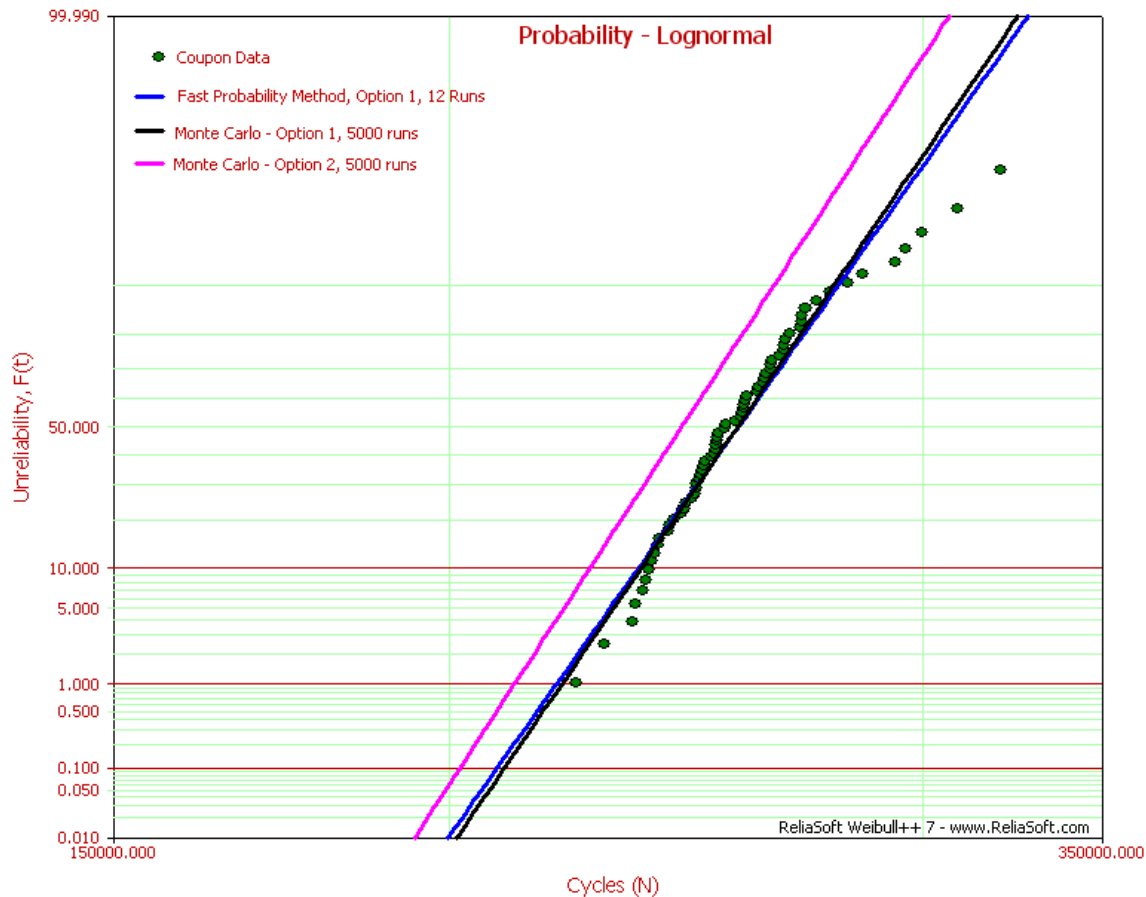




**Figure 13: Normal probability fit to 68 specimens for the Paris coefficient  $C$  and the correlation coefficient estimate between  $C$  and  $n$  is highlighted.**

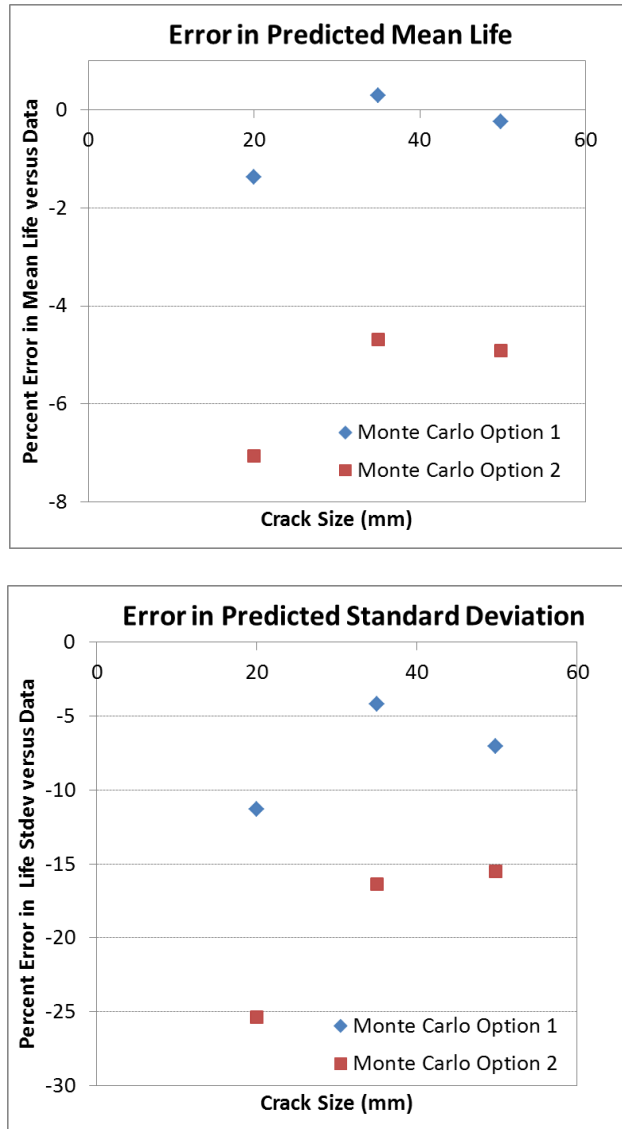
Since all 68 specimens were used in the estimation of the crack growth model coefficients – the life distribution from the model is expected to be close to the data. Life for crack to grow from an initial crack size of 9 mm to a crack size of 49.8 mm is reported in Figure 14. The log-normal distribution agreed well with the life distribution obtained through Monte-Carlo simulations and a comparison against the coupon data revealed that the option 1 approach provided life distributions that were very close to the experimental data. It was also observed that the results from option 2 method under-predicted the life significantly. The ability to use fast probability integration methods like the advanced mean value method to accurately represent the life variation with as few as 12 runs is also demonstrated through this benchmark. A significant element of the probabilistic framework is the ability to replace the Monte-Carlo simulation procedure by an efficient method that can compute distributions of life based on a few number of simulations. This is necessary when exercising the probabilistic framework on Figure 1 with finite element based fracture mechanics analyses from [2].

These results also demonstrate the deviation of the experimental life from log-normal at the right tail of the distribution. This deviation was the result of the intra-specimen variability reported in Figures 7, 8. If the two specimens with the highest crack size residuals (46 and 49) were removed from the data-set, it was noted that the experimental data followed a log-normal distribution in a better fashion and suggested that intra-specimen variability as the reason for the tail deviation in the data.



**Figure 14: Results of life for the crack to reach 49.8mm from an initial size of 9 mm using the estimates from option 1 and option 2. Note that the log-normal distribution is a good fit to the results from the Monte Carlo simulation except at the right tail.**

Figure 14 – option 1 method demonstrates the ability of the some of the elements of the probabilistic framework in terms of being able to accurately characterize the variability in fracture mechanics life through a modeling framework. It was also noted that the use of “+3 sigma material properties” (constant Paris  $n$  and +3 sigma  $C$  as noted in Figure 9) results in an estimate for minimum life of around 132,019 cycles that is significantly lower than the true -3 sigma life that is closer to 208,313 cycles. Figure 15 shows the error in the predicted life for the crack to reach 3 different crack sizes of 20mm, 35mm and 49.8mm in terms of the average (expected value) and standard deviation between the Monte Carlo simulations and the empirical life analysis of the experimental data. A comparison of the option 1 results against reference [5] demonstrates similar errors in mean life and standard deviation and significantly improved results over that published in reference [3,4] that use the option 2 approach. It should be mentioned that the use of a higher order crack growth model in reference [4] resulted in better estimates of mean life (within 2%) as opposed to the 5% errors that were noticed with the option 2 approach with the Paris Law Model in this work. The under-prediction of the standard deviation in the model is due to the presence of the 2 outliers (specimens 46 and 49) that are related to the intra-specimen variability in the data. Note that the 7% error in standard deviation in life at final crack size using option 1 is also smaller than the 10% errors noted in reference [4] and the 15% error in standard deviation in life at final crack size using option 2 is higher than the 10% errors noted in reference [4]. Since the “ $da/dN$ ” versus “ $\Delta K$ ” approach is reference [4] resulted in improvements over the option 2 approach presented here, it can be surmised that the use of a higher order crack growth rate model rather than the Paris model would potentially further improve the results with the option 1 approach presented here.



**Figure 15: Error in the predicted mean life and predicted standard deviation in life to a crack size as specified in the x-axis using the Monte Carlo method for options 1 and 2.**

The following conclusions can be made based on the results from this benchmark:

- Log-normal distribution was found to characterize the predicted life from the models as well as the experimental data in the absence of 2 outliers.
- The presence of intra-specimen variability in the experimental data was noted and was found to be a potential source of the deviation of the experimental life from log-normal at the right tails. Two specimens – 46 and 49 were identified as having a rather high intra-specimen variation compared to the Paris Law type crack growth.
- The use of “+ 3 sigma” property for C and constant n (as mentioned in Figure 9) to compute minimum life was shown to provide a much lower minimum life than the true – 3 sigma life from the experimental data.
- The high correlation coefficient between the Paris law coefficients was noted and recognized to be important in terms of capturing the variability in life accurately. This aspect has already been discussed in detail in [5, 7, 8].
- Estimation of crack growth model coefficients is performed more accurately in the “a versus N” space rather than the “da/dN versus “ΔK” space. Although computationally more



intensive, this approach is less sensitive to the numerical error introduced in the crack growth rate computation. This conclusion could have also been made by a comparison of results from reference [4] and [5] and these results explicitly emphasize that understanding.

- The ability to use fast probability integration methods to efficiently propagate (with 12 runs) the crack growth model parameter variation to compute the variation in life with the same level of accuracy as Monte Carlo methods (with 5000 runs) was shown here.
- Since the use of a higher order crack growth rate model in [4] resulted in improved results compared to the “da/dN versus “ΔK” estimation using the Paris Law model in this paper; the use of a higher order crack growth rate model that better captures curvature is expected to show an improvement with the option 1 approach and will be pursued in a future phase of this effort along with the implications of references [7, 8].

## V Extension to Industrial Applications

The results of this benchmark problem demonstrate some of the elements of the probabilistic fracture mechanics framework, however the extension to industrial applications (e.g. Aviation hot section components) as described in Figures 1, 2 require more challenging elements of the framework to come together as described here.

Challenges from a field validation and assessment perspective:

1. The validation of a probabilistic life assessment framework for component hardware (as opposed to coupon data) requires the understanding of the distribution of operating conditions since the life consumption in the field is a strong function of the operating conditions that impact the mechanical/thermal loading. A cumulative fracture mechanics life consumption model that accounts for the impact of the operational history using the full distribution of these parameters would be required (*The loading was deterministic/fixed in the benchmark problem considered in the paper*)
2. The ability to obtain good quality inspection data on components at regular intervals including information on crack size at the key failure locations is important from a validation perspective (*We get lower quality component field data compared with the quality of data we get on experimental specimens in the lab*)
3. The field life assessment often requires the assessment of both crack initiation life and crack propagation life since the field inspection data intervals may not purely track the crack propagation aspect. Although the crack propagation life may be a significant portion of the overall life of the component, the presence of an initiation life piece in the fielded hardware makes it a key element of the overall probabilistic life assessment framework (*The benchmark problem started with a constant initial crack size of 9mm with regular subsequent measurements of crack size*)

Challenges from a life modeling perspective:

4. Although crack growth model parameter variation is a significant source of scatter in life for both initiation and propagation; there are other sources of variation due to hardware geometry/manufacturing that need to be considered in the framework. These sources of variation can potentially impact the crack tip stress intensity to be a random variable.
5. Key sources of variation (e.g. operation, manufacturing, material) can be propagated through the life modeling process. However in a complex mechanical system, there are bound to be several sources of variation that can be difficult to characterize. A combination of field / experimental data from various sources for different modeling aspects (e.g. temperature, stress, life) needs to be integrated into the framework. The crack growth property variation is just one example of a source of variation that was captured in the current paper. A global sensitivity analysis framework accounting for the nature of the distributions is needed to identify the key input parameters that impact crack tip stress intensity, crack growth life and trajectory [12]. This screening is important since the number of input parameters that can potentially influence life can be large.

6. Computing the average life accurately in a complex mechanical system is challenging for many reasons (e.g. lab tests not representative of actual operating conditions, conservative methods that add margin used in the modeling phase during design, accuracy of modeling methods for advanced materials in harsh environments). These uncertainties (or lack of knowledge) also impact our ability to understand stress and temperature loading that is key to the computation of the stress intensity as well as the crack growth rate material properties. These uncertainties also limit our ability to propagate known sources of variation through the models to understand the variation in life. References [7, 8] highlight some of these challenges even in the context of the crack growth models considered for this benchmark.
7. Fracture mechanics analysis for complex components can be time intensive due to the use of finite element based approaches to obtain accurate stress intensity values for multiple crack geometries [2]. The ability to develop efficient and accurate (fast) meta-models that capture the non-linearity of the stress intensity response in the input parameter space is critical in the development of a probabilistic fracture mechanics simulation [13]

These are some of the key reasons for the differences that we often observe between the model predictions and the field data highlighted in Figure 2. Mismatch in the mean values between the model and the data help us understand modeling entitlement/uncertainty and provide the basis for improving the deterministic life models. Mismatch in the standard deviation values between the model and the data help us understand model based variation prediction entitlement and provide the basis for improving probabilistic life models. A preliminary framework for making these assessments has been developed in [14] and future improvements to address some of the challenges discussed here are currently being considered using a system level Bayesian approach [15, 16].

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